

SEMESTER-IV

COURSE 9: RING THEORY

Theory

Credits: 4

5 hrs/week

Course Outcomes

After successful completion of this course, the student will be able to

1. acquire the basic knowledge of rings, fields and integral domains
2. get the knowledge of subrings and ideals
3. construct composition tables for finite quotient rings
4. study the homomorphisms and isomorphisms with applications.
5. get the idea of division algorithm of polynomials over a field.

Course Content

Unit – 1

Rings and Fields

Definition of a ring and Examples – Basic properties – Boolean rings - Fields – Divisors of 0 and Cancellation Laws – Integral Domains – Division ring - The Characteristic of a Ring, Integral domain and Field – NonCommutative Rings - Matrices over a field – The Quaternion ring.

Unit – 2

Subrings and Ideals

Definition and examples of Subrings – Necessary and sufficient conditions for a subset to be a subring – Algebra of Subrings – Centre of a ring – left, right and two sided ideals – Algebra of ideals – Equivalence of a field and a commutative ring without proper ideals

Unit III: Principal ideals and Quotient rings

Definition of a Principal ideal ring(Domain) – Every field is a PID – The ring of integers is a PID – Example of a ring which is not a PIR – Cosets – Algebra of cosets – Quotient rings – Construction of composition tables for finite quotient rings of the ring Z of integers and the ring Z_n of integers modulo n .

Unit – 4

Homomorphism of Rings

Homomorphism of Rings – Definition and Elementary properties – Kernel of a homomorphism – Isomorphism – Fundamental theorems of homomorphism of rings – Maximal and prime Ideals – Prime Fields

Unit – 5

Rings of Polynomials

Polynomials in an indeterminate – The Evaluation morphism -- The Division Algorithm in $F[x]$ – Irreducible Polynomials – Ideal Structure in $F[x]$ – Uniqueness of Factorization $F[x]$.

Activities

Seminar/ Quiz/ Assignments/ Applications of ring theory concepts to Real life Problem /Problem Solving Sessions.

Text book

Modern Algebra by A.R.Vasishta and A.K.Vasishta, Krishna Prakashan Media Pvt. Ltd.

Reference books

1. A First Course in Abstract Algebra by John. B. Farleigh, Narosa Publishing House.
2. Linear Algebra by Stephen. H. Friedberg and Others, Pearson Education India
